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CYLINDRICAL DISTRIBUTION OF NODAL STRENGTH AROUND THE PIPE NORMAL*

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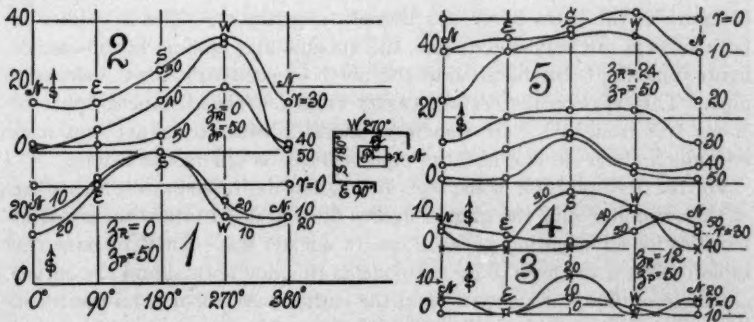
1. *Circular and Radial Survey.*—The unexpected non-symmetric results obtained in the last paper, induced me to try a group of experiments relative to the center (nearly) of the table. The resonator was therefore carried in a circle of radius r_R around the plumb line let fall from the mouth of the pipe. The measurements were then repeated for successive levels of the pipe (z_P = 40 cm. and 50 = λ cm. nearly) and of the resonator (z_R = 0, 12 = $\lambda/4$, 24 = $\lambda/2$ cm.). The relation of the different locations is suggested by the inset of figures 1, 2, which is a reduced plan of the room, with the pipe in the vertical above P on the table. The directions + y (west) and − x (south) are thus toward walls, whereas + x (north) and − y (east) are largely toward open spaces from the table.



In the first exploration (here omitted) the mouth of the pipe P was z_P = 40 cm. above the table. Carrying the mouth of the resonator on the table with its axis kept parallel to x, around successive circles of radius r_R

= 0, 10, 20, 30, 40, 50 cm. gave intensity distributions, s, very definite and inter-related, but quite unsymmetrical to the pipe vertical. The s-oscillations in eastern and northern R-displacements from the center, (i. e. to open locations) were meager, while the s-oscillations for southern and western directions, trending toward closed parts of the room, were very marked. As r increased from 0 to 10, 20, 30, the maximum intensity lay toward the south; beyond this (r = 40, 50 cm.), it moved to the west. All the maxima of intensity were larger than the intensity on the table, under the pipe (r = 0); even at r = 50 cm. (meaning a pipe resonator distance of 64 cm.), the s-value was a little in excess of s = 23 for the normal case.

Another view of the results was given by collecting all the points on the line S-N along x, and on the line W-E along y. The features of the latter were somewhat vague, but the S-N line presented conspicuous crests and troughs. These were from 40 to 45 cm. apart, and situated on either side of, but without other appreciable reference to the pipe vertical P in their contours. The inference would be that we must look to other causes for their occurrence.

The mouth of the pipe was now raised in the same vertical to z_P = 50 cm. above the table, or a little over a wave-length (48 cm.). The data so obtained are constructed in the way described in figures 1, 2, for the rotation of the resonator and figure 6, for its corresponding $\pm x$ and $\pm y$ displacement. The central intensity (r = 0) has increased, because of the λ-elevation conformably with the earlier results. The curves as a whole are simpler in outline; the unique maximum is again transferred from S to W as r increases, this time a little sooner, in fact; i.e., before r reaches 30 cm. For large values of r, intermediate azimuths would have been desirable; but as the phenomena are adequately given by the linear surveys of figure 6, the extra labor was thought superfluous. The S-N lines in both surveys are very much alike, the maxima and minima in the case of figure 6 having been shifted into the north or positive x, for the elevated pipe. This also recalls certain earlier results. The distances apart of crests and troughs is here somewhat more difficult to define; but it is still much above 40 cm. and the pipe vertical is ignored as before.

For the fixed position of the pipe, it was now desirable to raise the plane of the resonator and the graphs figures 3, 4, and 5 contain the nodal intensities for an elevation z_R = 12 cm. (a quarter wave-length) above the table. This is the level of an antinode at the pipe normal and the curves are therefore low throughout, as in the earlier work for normal positions. Nevertheless as we depart from the pipe normal or center, to circumferences of radii 10, 20, 30 cm., the unique maxima in the south become prominent and beyond r = 30, they again shift to the west.

In figure 7, the S-N line has the same characteristics as in figure 6 with

its two marked crests. Somewhat less certainly the resemblance in W-E lines may be detected. As in all the N-S lines the central trough lies for some reason to the right of the pipe normal. The maxima are far apart apparently.

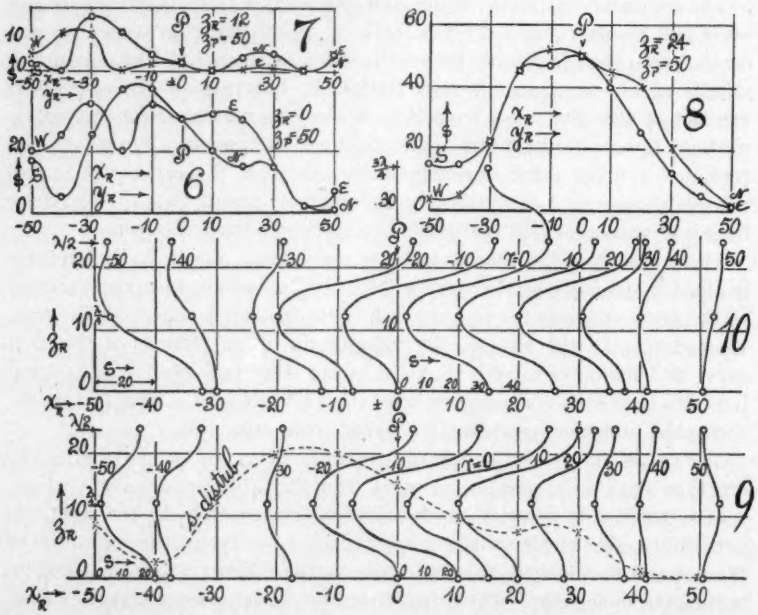
The resonator was finally transfered to a level, $\lambda/2 = 24$ cm. above the table and moved around the pipe axis, its mouth lying on successive circumferences of radii r, with the results given in figure 5. The maxima of all the curves are here curiously flattened; but there is still a gradual transfer of the chief crest from S to W, as r increases. Probably intermediate observations would have modified the curves. Since $\lambda/2$ determines a nodal plane normally below the pipe, the intensities are all relatively large, but they fall off as a whole so rapidly with r, that there is no intersection and all curves may be given in a single diagram.

The graphs showing the intensity, s, along the S-N and W-E directions in figure 8, exhibit a total change of form. The two crests have seemingly all but coalesced near the pipe normal. Owing to an accident which somewhat dislocated the fringes, the indentation in the middle of the S-N curve and the bulge toward the right in the other may need modification but this is of little consequence here, since the plane at a level of $z = 24$; above the table contains a single marked crest, only.

2. *Vertical Survey.*—It is now of interest to bring the values of intensity together when the resonator moves up from the table on the same vertical. As constructed in figure 9, with s laid off horizontally to the right, z_R vertically and with the origin of *each* tripoint s-curve at the x position of the vertical to which it belongs, these vertical distributions of nodal intensity strike the eye. The intensities s on the table for the same x positions are also sketched in. Under the pipe at $z_P = 50$ cm., the nodal surface at the table rapidly decreases in s-intensity to the antinode at $z_R = \lambda/4$; s then rapidly increases again to the more pronounced nodal surface at $z_R = \lambda/2$, nearer the pipe. Toward the south however the distinction between node and antinode is not regularly sustained: at $x = -20$ cm. the antinode has gained in strength and at $x = -30$ cm. the strength of nodes and antinodes does not differ much. Beyond this ($x = -40, -50$), however, nodes and antinodes are again sharply contrasted. Toward the north from the pipe normal ($x = 0$), these differences die out more gradually, until at $x = 50$ cm. the intensities are for some further reason, reversed.

What is very surprising, however, is the observation that throughout the whole of the extent of a linear meter ($x = \pm 50$) symmetrically to the pipe normal, the nodal and antinodal planes at $z_R = 0, \lambda/4, \lambda/2$, remain parallel to the table, so far as can be seen. The only distinction within this stretch is the distribution of nodal intensity as exhibited in the continuous curves sketched in, or separately in figures 6, 7 and 8. It would

therefore here seem to be a mistake to associate wave-length with crests and troughs in these diagrams, however plausibly such inferences may obtrude. It is very much more probable that in the south some extraneous s-distribution has been superposed.



The West-East distributions ($\pm y$) though not so sharp in their features in figures 6, 7 and 8, show an equally pronounced character when treated in the same way relative to vertical variations, in figure 10.

Each of the triplet graphs is to be referred to an origin at the x_R marked on the curve and the s-intervals are 20 scale-parts, as heretofore. Since the edges of the table are at $y = \pm 60$ cm. (nearly), the nodal plane does not extend quite so far; but it does not follow that these edges have much influence on the results, since the same kind of graphs appear in figure 9, where the edges of the table are nearly 50 cm. further. The degradation of the nodal plane occurs first at $y = 50$ cm. in the east as it did in the north.

In the west it is still strong at $y = -40$, and now promises to run through another cycle of intensity as in the previous case. The antinode very nearly $s = 0$ throughout the east, begins to lose identity between $y = -20$ and -30, to be followed by oscillations. Finally, the first nodal plane above the table degenerates here more obviously than before, between $y = -30$ and -40.

The records in figures 9 and 10 contain an almost complete example of a free nodal region sustaining itself $z_R = 24$ cm. above the table, and figure 8 gives the acoustic pressures within it. These are largest near the pipe, though not quite symmetric and it is possible that the direct ray may here influence the resonator. In general, however, i.e., at greater distances, the direct ray is relatively ineffective and it is to the occurrence of nodal regions alone that the resonator then responds, even when the direct ray would seem to have the advantage of nearness. Thus in figure 10, the curve for $r = 0$, when tested at the second antinodal plane ($z_R = 36$ cm.), although the pipe is now but 12 cm. away, nevertheless retains the features of a pronounced minimum.

* Advance note from a Report of the Carnegie Inst., of Washington, D. C.

IONIZATION AND ABSORPTION EFFECTS IN THE ELECTRIC FURNACE

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In 1914, the publication by Fowler of the series formulae for enhanced lines showed that the Rydberg constant "N," occurring in the formulae for arc lines, is changed to 4N for enhanced lines. This established the view based on a very general difference of behavior in both laboratory and celestial sources, that the enhanced lines and the arc lines arise from differently constituted emitting centers. The condition pictured by Bohr's theory is that the enhanced lines are produced by atoms which have lost one electron, while the normal atom emits the arc lines.

Recent work by Saha applies the relations recognized in physical chemistry to the case when the normal atom is changed into an ionized atom with an electron set free. A definite equilibrium is considered as established, represented in the case of calcium by $Ca \rightleftarrows Ca^+ + e$.

It follows from this that if we add a large supply of electrons from some substance which becomes ionized more readily than calcium, fewer electrons from the calcium can exist in the free state, and the proportion of Ca^+ atoms becomes less, with a corresponding weakening of the enhanced lines relative to the lines of the normal atom.

In the electric furnace, the enhanced lines of some substances can be produced faintly, and the effect of a mixture with an easily ionized substance may be tested, under the same conditions of temperature and pressure for the mixture as for the pure substance. Experiments have been

carried out in which both emission and absorption spectra were compared in this way.

The spectra of calcium and strontium showed in each case a decided weakening of the enhanced lines when a mixture of either element with potassium or caesium was examined, while the arc lines, presumably due to the unionized atoms, were unaffected. Barium, with a smaller difference between its ionizing potential and that of the added substance, showed a less pronounced effect. Sodium, whose ionization potential is nearly the same as that of barium, produced no weakening of the barium enhanced lines. The experiments thus support the view of an equilibrium between electrons and ionized atoms, which may be disturbed by an added supply of electrons.

Certain features of absorption spectra may be described briefly. By holding the furnace at a measured temperature and passing through the tube a beam of white light from a tungsten filament the absorption spectrum may be readily produced. Turning off the lamp enables the emission spectrum of the vapor at the same temperature to be photographed for comparison.

The iron spectrum under these conditons showed no decided difference in emission and absorption except as to the production of the ultra-violet spectrum. The low temperature stage (1600°–1700° C.) ended in emission at λ 3440, except that a few scattered lines as far as λ 3000 could be brought out faintly. The continuous spectrum when a plug is used in the tube at the same temperature has about the same limit, and it has been a general experience that the emission of vapors in the furnace extends no farther than the spectrum of a black body at the same temperature.

The possibility remained that metallic vapor too cool to emit in the far ultra-violet could still show absorption lines if a sufficiently hot source of white light in a chamber transparent to the ultra-violet were available. Lacking such a source, the light from exploding wires was tried, which Anderson has shown to give a continuous spectrum extending to short wave-lengths. Dr. Anderson kindly operated the explosion apparatus for this experiment, the best results being obtained with wires of lead. With the furnace temperature as low as 1600° C., absorption lines of iron could be identified as far as λ 2298, a limit set by the sensitiveness of the photograhic emulsion. These lines had previously been obtained as far as λ 2447 in the high temperature furnace when the continuous spectrum was supplied by a graphite plug in the tube. We thus have the condition that for the emission spectrum a definite limit is set by the black body radiation at that temperature, but that the vapor is able to absorb radiation of much shorter wave-length.

One feature of Saha's theory is that a vapor too cool to emit should be able to absorb the lines of the principal series. This was tried with the

strong flame lines λλ 4227, 4607, and 5535 of calcium, strontium, and barium, respectively. In each case a temperature was found at which no emission could be detected, but when white light was sent through the furnace the lines appeared distinctly in absorption. For these favorable lines, the view of Saha is thus confirmed. It will be of interest to test other series lines in the same way.

In previous investigations on the absorption of vapors of the alkali metals, the lines of the principal series have appeared, but not those of the subordinate series. Since in emission spectra the latter require higher temperatures than the principal series lines, it seemed probable that a relatively high temperature was required to produce them in absorption. Experiment showed this to be the case. Sodium, potassium, caesium, and rubidium were tested by heating to a high temperature in a tube having a plug to give the continuous ground. The subordinate series lines appeared in absorption for each element.

The general character of the absorption phenomena in the furnace may be described as in general a reversal, when white light is passed through the tube, of the emission spectrum at the same temperature. Exceptions are found in the easily excited principal series lines and perhaps others, which the vapor can absorb before it emits, and in the ultra-violet region beyond the limit, at the given temperature, of the black-body radiation. In the latter case an absorption spectrum appears beyond the emission limit when a sufficiently hot source of white light is employed.

PRELIMINARY NOTE ON A BIOMETRICAL STUDY OF THE RELATIONS OF CERTAIN VISCERA IN TUBERCULOSIS[1]

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One of the most fundamental problems of biology is that of adaptive regulation to the end of maintaining the life of the individual. The ability to readjust the functional relations of the parts of the organism, after they have been in any way disturbed is widespread among living things generally, including man. A well-known example is seen in the functional hypertrophy of the kidney. If one kidney is removed the remaining one promptly enlarges and carries on the work formerly divided between the two. Many similar examples might be cited.

We have been engaged for the past year in a study of this phenomenon of adaptive functional regulation from a somewhat new point of view. Starting from the justifiable assumption that the records of pathological anatomy, as set down in autopsy protocols, should if adequately analyzed

give a picture of regulatory processes perhaps more detailed and definite than is likely to be found elsewhere, it seemed probable that quantitative study of the ratios existing between the weights of the more important viscera would indicate with great exactness any disturbances in the normal balance of the parts of the organism. The results, we believe, fully bear out this assumption. This first investigation, which has just been completed, was restricted entirely to tuberculosis as the pathological condition involved. This choice was deliberate and based upon the known profound constitutional effects of the disease. The data studied are six organ-weight indices or ratios, defined as follows:

$$\text{Index A} = \frac{\text{Liver weight}}{\text{Heart weight}} = \text{L/H}$$

$$\text{Index B} = \frac{\text{Liver weight}}{\text{Spleen weight}} = \text{L/S}$$

$$\text{Index C} = \frac{\text{Liver weight}}{\text{Right kidney weight} + \text{Left kidney weight}} = \text{L/K}$$

$$\text{Index D} = \frac{\text{Heart weight}}{\text{Spleen weight}} = \text{H/S}$$

$$\text{Index E} = \frac{\text{Right kidney weight} + \text{Left kidney weight}}{\text{Heart weight}} = \text{K/H}$$

$$\text{Index F} = \frac{\text{Right kidney weight} + \text{Left kidney weight}}{\text{Spleen weight}} = \text{K/S}$$

These indices were calculated from the data given on the protocols of such of the first 5000 autopsies (1341 in all) of the Johns Hopkins Hospital as revealed tuberculous lesions, and at the same time conformed to certain other requirements. The material was divided for purposes of study by race, sex, activity or quiescence of lesion at time of death, age, and character of lesion.

The age characteristics of this autopsied, tuberculous, hospital population were compared in detail with those of the general population, and with that part of it dying of tuberculosis. In general there is a close agreement in mean age at death between the autopsied hospital population showing active tuberculous lesions, and the portion of the general population dying of tuberculosis.

One of the most interesting and novel results of the study has been to show that certain of these organ-weight indices change with the age of the individual. These changes may be looked upon as phenomena of senescence beginning very early in life, indeed practically at birth. The detailed results as to correlation with age may be briefly summarized as follows. There is a significant *negative* correlation in all groups between the value of Index A and age. This means that as age advances the liver/heart ratio tends to take on lower and lower values. Index B is, on the whole

not significantly correlated with age. The same is also true of Index C. With the exception of the white groups with active tuberculous lesions, which show no sensible correlations of this index with age, there is generally a significant *positive* correlation of Index D with age. Index E exhibits a relatively high, and in general certainly significant *negative* correlation with age. Index F is not correlated with age. Inasmuch as the material included all ages from infancy to extreme old age, these correlational results with age have a general biological significance. They indicate that, so far as the present material may be trusted to portray general relationships, the functional balance of certain organs, notably the heart and spleen, changes throughout life in an orderly manner, capable of expression by mathematical equations. Such regression equations, which are throughout linear in character, were computed for all cases where the correlation coefficients were significant, and made the basis of later corrections of the biometric constants of the indices, to allow for the influence of age.

A detailed study of the relation of race (white or colored) to the values of the several organ weight indices, after proper corrections had been made for the influence of age, leads to the general result that where there are no lesions in either of the organs involved in an index and where the number of cases is large enough to give reliable results, there is, broadly speaking, no difference between white and colored racial groups in either mean or variability, except in the case of Indices B and D. In these cases it is only the means and not the relative variabilities that differ. In both cases it is the colored group that has the higher value for the indices.

In general there are no significant differences between the sexes in either mean values or variability of the indices, after proper corrections have been made for the influence of age. In these organ-weight indices we are evidently dealing with fundamental functional characteristics of the organism, which express in a hitherto unnoted way the extraordinary regulatory powers which are in a profound manner associated with the maintenance of life. The absolute weights of the several viscera may differ widely in the different races and sexes, but the ratios of these same viscera, in respect of weight, appear from the present experience to have a constancy and stability biometrically, which, broadly speaking, entirely transcends the influence of race and sex.

After making proper corrections for differences in the age distributions of the groups involved, the question of the influence of tuberculosis *per se* upon the organ-weight indices was attacked. This problem was approached in several ways of which the most significant was to compare two groups of which one contained only cases wherein the *sole* significant lesions at death were those of tuberculosis, while the other contained cases where in addition to the tuberculous lesions present (active or inactive) were

other lesions in themselves sufficiently grave to have caused death, had there been no associated tuberculosis. On the basis of this grouping it is found that five of the six indices (namely all those involving heart or spleen or both) show large and significant differences as between the two groups. These differences everywhere are of the sort which would arise if the effect of fatal tuberculosis was to lower the absolute weight of the heart and increase that of the spleen. The significance of these results, as well as that of the differences in mean age at death in the two groups will be discussed in the detailed paper, which will appear in the Reports of the Johns Hopkins Hospital.

The work is being continued with other pathological groups, and it is believed will become more interesting and significant as we become able to compare the results from different types of lesions.

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 58.

THE EFFECT OF ETHER UPON THE MIGRATION OF THE SCALE PIGMENT AND THE RETINAL PIGMENT IN THE FISH, FUNDULUS HETEROCLITUS

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When an adult Fundulus is etherized the pigment in the scale melanophores shows a complete distal migration and the fish becomes dark in color. In this respect ether produces an effect the reverse of adrenalin. But adrenalin not only causes a proximal migration of the scale pigment; it induces a distal migration of the retinal pigment (Gilson, '22). Does ether also influence the retinal pigment and, if so, is its effect on that pigment the reverse of that of adrenalin? To test this question Fundulus was etherized either in the dark or in the light and its eyes prepared for study. Fish that had been some time in the light were etherized by allowing a stream of 5% ether to drip upon the gills from ten to fifteen minutes. Before etherization the scale melanophores showed a complete proximal migration of the pigment granules. Within one to four minutes after the application of ether had begun the melanophores were in a state of complete distal migration. At the end of ten or fifteen minutes the animals were killed and the eyes were prepared. The same procedure was carried out upon fish that had been kept in the dark. 5% ether was also applied to both light and dark fish by immersing the body as far as the gills in the ether solution for twenty or thirty minutes after which their eyes were prepared. The scale melanophores of light fish which were immersed

in this way showed a distal migration in from five to fifteen minutes wherever the ether solution touched the skin. As controls in each experiment eyes of unetherized fish that had been kept either in the light or in the dark were prepared.

The condition of the retinal melanophores of fish which had been etherized, either by applying the solution to the gills or to the body, differed in no way from those of normal fish kept under the same conditions of illumination. Those of light fish showed a distal and those of dark fish a proximal migration of the pigment granules. Ether was applied directly to the eyes by immersing excised eyes in a 5% solution for thirty minutes followed by immediate fixation. Very commonly the pigment in eyes which were treated in this way was extremely contracted and broken up into rather large round masses separated from each other by spaces much larger than any seen in normal eyes. This was probably a post mortem change due to the disintegration of the pigment cells. In eyes which appeared normal, however, the condition of the pigment was not different from that of retinas which had not been treated with ether. The post mortem change described above was often seen in eyes whose optic nerves had been cut before the application of ether. Here too the pigment cells did not seem to be affected at all by the ether.

Arey ('16) found that carbon dioxide and ether, both in darkness and in light, and in excised as well as in undisturbed eyes, completely check the movement of all the retinal elements in fishes. The retinal pigment of fishes anesthetized under one condition of illumination and then removed to another remained in the condition characteristic of the first state of light or darkness. Hence I conclude that ether does not have an action on the pigment cells of the retina in fishes the reverse of that of adrenalin. It merely arrests the retinal pigment in whatever condition it happens to be when the drug is administered.

An explanation for the differing effects of adrenalin and ether upon the scale and retinal melanophores in fishes may probably be found in the widely different methods of control of the two types of melanophores. The scale melanophores are controlled by the sympathetic nervous system and in the light they are held in the contracted condition by means of a tonus established by impulses in the central nervous system set up by light stimuli received through the eyes. The contracted state, then represents a state of stimulation. Spaeth ('16) thinks that melanophores are a physiologically modified type of smooth muscle tissue. Adrenalin, the effect of which upon smooth muscle tissue is comparable to sympathetic stimuli, causes a contraction of scale melanophores. Ether anesthetization removes the nervous tonus and allows the melanophores to expand. The melanophores of the retina, however, are not controlled by means of 'retino-motor' nerve fibres (Arey, '16, p. 180), but their activities are simply

"expressions of protoplasmic responses to definite stimulating agents." The effect of adrenalin is the expression of the action of such an agent from the blood. Ether inhibits the activities of the melanophores arresting them in whatever condition they may happen to be.

Arey, L. B., 1916. The movements in the visual cells and retinal pigment of the lower vertebrates. *J. Comp. Neurol.*, **26** (121–190).

Gilson, A. S., 1922. The diverse effects of adrenalin upon the migration of the scale pigment and the retinal pigment in the fish, Fundulus heteroclitus, Linn. *Proc. Nat. Acad. Sci.*, **8** (130–133).

Spaeth, R. A., 1916. Evidence proving the melanophore to be a disguised type of smooth muscle cell. *J. Exp. Zoöl.*, **20** (193–215).

THE DIVERSE EFFECTS OF ADRENALIN UPON THE MIGRATION OF THE SCALE PIGMENT AND THE RETINAL PIGMENT IN THE FISH, FUNDULUS HETEROCLITUS, LINN

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Bigney ('19), working with the frog, found that the injection into the blood stream of this animal of small quantities of adrenalin caused a proximal migration (1) of the granules in the dermal melanophores of the skin and a distal migration of the granules in the melanophores of the retina. The present investigation was undertaken to determine if similar effects were to be found in fishes.

For these experiments, the animal used was the common kilifish, Fundulus heteroclitus Linn. This fish shows a marked response, both in the scale (dermal) and the retinal melanophores to light and to darkness. When it is placed in the light over a white background, it shows a proximal migration of the scale melanophore granules and a distal migration of the retinal melanophore granules. In the dark, these conditions are reversed, the scale melanophores showing a distal and the retinal melanophores a proximal migration.

The method adopted for stating the effects of the adrenalin upon the retinal pigment was that of expressing the width of the retinal pigment extension, as measured along processes showing maximal extension in the region measured, as a fraction of the total distance from the outer boundary of the pigmented epithelium to the external limiting membrane, this distance being taken as 100. For purposes of brevity in this paper, the terms *light fish* and *dark fish* will be used to indicate fish which have been kept in the light and fish which have been kept in the dark, respectively. In the typical light fish, the pigment shows an average extension of 83 units on this scale. Such light retinas show, in almost all cases, a secondary

concentration of pigment between 70 and 80. In dark animals, the average extension of the retinal pigment is 59, the pigment being massed at the outer region of the retina.

Retinas were examined of normal light fish, of normal dark fish, and of light and dark fish some of which had received intramuscular, some intra-abdominal injections of physiological salt solution, of adrenalin 1:1,000, or of adrenalin 1:10,000. The adrenalin used was that prepared by Parke Davis and Company and sold as "Adrenalin 1:1,000." The adrenalin was allowed to act for varying times before the retinas were fixed, with the results that the greatest effect was found to occur in those dark animals which had been injected with 0.2 cc. of adrenalin 1:1,000, and between thirty minutes and one hour after injection.

Of the animals to be experimented upon, those to be used as dark animals were kept in a photographic darkroom for at least sixteen hours before being treated; those to be used as light animals were placed in a white dish which was kept for at least a day by a north or west window and the fish were treated and killed between one and three o'clock in the afternoon. The animals were killed by quick decapitation with a pair of scissors, the head was opened longitudinally, and immersed in Bouin's picro-formol mixture. Fixation was allowed to proceed for about fifteen hours. The eyes were then dissected free from the head, washed thoroughly in 70% alcohol and dehydrated in 90% and absolute alcohol. While in absolute alcohol, the cornea of each eye was sliced off with a razor and the lens removed with a pair of fine forceps. The eyes were then cleared in xylol, infiltrated and imbedded in paraffin and serial sections were cut, in most cases 7 micra in thickness, and in a plane perpendicular to the face of the eye and through the optic nerve. Measurements could thus be made on radial sections at a point about half way from the entrance of the nerve to the outer rim of the retina. For measuring, a high dry objective and camera lucida were used, projection being made upon a line ruled along a sheet of paper and upon which the actual measurement was made. All experiments were performed at room temperature.

The results obtained are summarized in *Table A*. The values given in the table are the averages of all experiments for each class, plus or minus the probable error (P. E.) as obtained by the use of Bessel's formula. This probable error indicates the relative value of the results by taking into account not only the standard variation of each series but also the number of experiments performed.

It will be seen that when light animals are injected with 0.2 cc. of adrenalin 1 : 1,000, there is no *significant* change in the width of the pigmented region of the retina. In dark animals which had been injected into the body cavity with 0.2 cc. of adrenalin at the strength of 1:1,000, there was shown a marked change of the pigment position, the pigment

TABLE A.

Distance from outer boundary of pigmented epithelium to external limiting membrane	100
Extent of retinal pigment processes in:	
Normal Light Fish	82 ± 2
Normal Dark Fish	59 ± 3
Light Fish, 0.2 cc. Adrenalin 1 : 1,000 intra-abdominally	80 ± 1
Dark Fish, 0.2 cc. Physiological salt solution, intra-abdominally	64 ± 1
Dark Fish, 0.2 cc. Adrenalin 1 : 1,000 intra-abdominally	77 ± 2
Dark Fish, 0.2 cc. Adrenalin 1 : 10,000 intra-abdominally	66 ± 3
Dark Fish, 0.2 cc. Adrenalin 1 : 1,000 intra-muscularly	74 ± 11
Dark Fish, 0.2 cc. Adrenalin 1 : 10,000 intra-muscularly	68 ± 3

Average weight of fish = 4.3 grams

$$\text{P. E.} = 0.6745\frac{\Sigma d^2}{n(n-1)}$$

where d is the deviation of any observation from the mean and n is the number of experiments in the series.

processes having an average extension of 77. Qualitative examination of these retinas showed a marked tendency toward the light condition. In these animals, however, the secondary, distal concentration of pigment was usually lacking. Injection into the abdominal cavity of weaker adrenalin solution, and injection into the dorsal musculature gave generally unsatisfactory results. Dark animals into whose body cavity physiological salt solution had been injected showed a barely significant change in the condition of the retinal pigment. Whether this very slight change is due to the effect of the physiological salt solution itself, or to the secretion of adrenalin by the animal remains to be determined. In all cases, the scale melanophores of animals injected with adrenalin showed the extreme proximal migration of the melanophore granules which has been described by previous workers.

Klett ('08), working with the frog, and using only light animals, was able to discover no effect upon the retinal pigment when a solution of adrenalin was injected into the blood stream. Injection of a very strong solution of adrenalin into the anterior chamber of the eye, however, caused a contraction of these melanophores. Bigney believes, and probably rightly, that this effect was due to the toxic nature of the solution used. Fujita ('11) used dark frogs and found that the injection of adrenalin into the bloodstream caused a distal migration of the retinal pigment. This finding was confirmed by Bigney.

Experiments upon Fundulus, here reported, demonstrate that the injection of adrenalin will cause a distal migration of the pigment processes in the retinas of dark animals, but produces no significant change in the already extended processes of light animals. The changes are of a reverse nature in the scale melanophores, the injection of adrenalin into the animal

causing a marked proximal migration of the granules. These findings are thus in accord with those of Fujita and Bigney in their work upon the frog, and opposed to those of Klett, also upon the frog.

Arey, L. B., 1916. "The Movements in the Visual Cells and Retinal Pigment of the Lower Vertebrates." *J. Comp. Neur.*, **26** (121–190).

Bigney, A. J., 1919. "The Effect of Adrenin on the Pigment Migration in the Melanophores of the Skin and in the Pigment Cells of the Retina of the Frog." *J. Exp. Zoöl.*, **27** (391–396).

Fujita, H., 1911. "Pigmentbewegung und Zapfenkontraktion im Dunkelauge des Frosches bei Einwirkung verschiedener Reize." *Arch. vergl. Ophthalm.*, Jahrg. **2** (164–179).

Klett, 1908. "Zur Beeinflussung der phototropen Epithelreaktion in der Froschretina durch Adrenalin." *Arch. Anat. Physiol.*, Jahrg., 1908, *Physiol. Abt., Suppl. Bd.* (213–218).

[1] Arey ('16) used the terms *distal* and *proximal* to designate the migration of the pigment granules away from the center of the melanophore cell and toward that center respectively. Hence a proximal migration would bring about what is ordinarily called the contracted condition of the melanophore and a distal migration the expanded condition.

A SIDE LINE IN THE IMPORTATION OF INSECT PARASITES OF INJURIOUS INSECTS FROM ONE COUNTRY TO ANOTHER

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Since the extraordinary initial success of the Department of Agriculture in importing the Australian ladybird beetle (*Novius cardinalis*) into California to destroy the fluted scale in the eighties, a great deal of successful work of the same sort has been done in different parts of the world. Most of these attempts have been made in a haphazard sort of way and a number of them have succeeded to a notable degree in spite of the rather unscientific way in which the importations were made and the lack of competent scientific study before the importations were attempted. The one thing which seems to have been rather carefully guarded from the start is the possibility of importing secondary parasites which might destroy the desirable assisted immigrants. Until recently there have seemed to be no time and no especial necessity for a careful biological study of the imported parasites before the importation has been attempted. Now in many cases such preliminary studies seem to be very necessary and they are being made.

One point which has undoubtedly resulted in the non-establishment of imported species has been the fact that in their native homes they had alternate hosts, feeding during one generation upon one species and during another generation upon another. Frequently the alternate host or a vicarious species has not existed in the country into which the parasites have been introduced.

On the other hand, such secondary and supplementary hosts have in a number of cases existed there, and as a result the establishment of the imported species was not only rendered more certain but the species has acted in a most beneficial way by destroying other pests, some of them in fact being native to the country into which the imported parasite was brought.

It is this side line in such importations upon which I wish to dwell for a moment in this paper.

Parasite introduction has never been attempted in any part of the world on so large a scale as it has been in this country since 1905 in the effort to secure the European and Japanese parasites and natural enemies of the gipsy moth and the European enemies of the brown-tail moth. In this work, although more than thirty species have been imported, not more than seven or eight have become established. Lack of secondary hosts may have been the cause of the failure of many of them, or there may have been other causes. An attempt is being made at the present time to determine these causes, and for the first time since the world war experts from the Bureau of Entomology are now in Europe and Japan studying the native parasites of the gipsy moth and endeavoring to send over new supplies of those species which previously failed of establishment and at the same time to secure additional species which we had not found before the war.

But the value of the comparatively few species that have actually become a part of the insect fauna of the United States as the result of the earlier introductions has been very considerably enhanced from the fact that some of them have taken readily to other imported or native caterpillars which destroy American trees and crops. The following table indicates the present situation with regard to four of these species.

HYMENOPTEROUS PARASITES

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
Euproctis chrysorrhoea L.	*Apanteles lacteicolor* Vier.	*Datana ministra* Drury	Rare
	Apanteles lacteicolor Vier.	*Hyphantria textor* Harris	Rare
	Apanteles lacteicolor Vier.	*Apatela hasta* Guenée	Rare
	Apanteles lacteicolor Vier.	*Schizura unicornis* S. & A.	Only in the laboratory, but here readily attacked
Euproctis chrysorrhoea L.	*Meteorus versicolor* Wesm.	*Hemerocampa leucostigma* S. & A.	Rare
	Meteorus versicolor Wesm.	*Hyphantria textor* Harris	Rare
Porthetria dispar L.	*Apanteles melanoscelus* Ratz.	*Hemerocampa leucostigma* S. & A.	Apparently abundant

DIPTEROUS PARASITES

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
Porthetria dispar L., and *Euproctis chrysorrhoea* L.	**Compsilura concinnata* Meig.	*Stilpnotia salicis* L.	Common and important
	**Compsilura concinnata* Meig.	*Basilarchia archippus* Cramer	Common and important
	**Compsilura concinnata* Meig.	*Euvanessa antiopa* L.	Common and important
	**Compsilura concinnata* Meig.	*Vanessa atalanta* L.	Common and important
	**Compsilura concinnata* Meig.	*Aglais milberti* God.	Common and important
	**Compsilura concinnata* Meig.	*Diacrisia virginica* Fab.	Common and important
	**Compsilura concinnata* Meig.	*Datana ministra* Drury	Common and important
	**Compsilura concinnata* Meig.	*Euchaetias egle* Drury	Common and important
	**Compsilura concinnata* Meig.	*Estigmene acraea* Drury	Common and important
	**Compsilura concinnata* Meig.	*Hemerocampaleucostigma* S. & A.	Common and important
	**Compsilura concinnata* Meig.	*Cirphis unipuncta* Haw.	Common and important
	**Compsilura concinnata* Meig.	*Papilio polyxenes* L.	Common and important
	**Compsilura concinnata* Meig.	*Papilio turnus* L.	Common and important
	**Compsilura concinnata* Meig.	*Anosia plexippus* L.	Common and important
	**Compsilura concinnata* Meig.	*Apatela hasta* Guenée	Common and important
Porthetria dispar L., and *Euproctis chrysorrhoea* L.	**Compsilura concinnata* Meig.	*Apatela brumosa* Guenée	Common and important
	**Compsilura concinnata* Meig.	*Schizura concinna* S. & A.	Common and important
	**Compsilura concinnata* Meig.	*Polygonia interrogationis* Fab.	Common and important
	**Compsilura concinnata* Meig.	*Papilio troilus* L.	Common and important
	**Compsilura concinnata* Meig.	*Alypia octomaculata* Fab.	Common and important
	**Compsilura concinnata* Meig.	*Epargyreus tityrus* Fab.	Common and important
	**Compsilura concinnata* Meig.	*Mamestra adjuncta* Boisd.	Apparently important

*Besides the above listed hosts we have reared *Compsilura* from about 60 other species of native larvae. We also have records of several more species reared by others.

DIPTEROUS PARASITES (Cont'd)

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
	**Compsilura concinnata* Meig.	*Notolophus antiqua* L.	Apparently important
		Croesus latitarsus Nort.	Apparently important
	**Compsilura concinnata* Meig.	*Phlegethontius quinquemaculata* Haw.	Apparently important
		Pontia rapae L.	Apparently important
	**Compsilura concinnata* Meig.	*Malacosoma americana* Fab.	Rare
	**Compsilura concinnata* Meig.	*Malacosoma disstria* Hübn.	Rare
	**Compsilura concinnata* Meig.	*Hyphantria textor* Harris	Rare
	**Compsilura concinnata* Meig.	*Mamestra picta* Grote	Rare
	**Compsilura concinnata* Meig.	*Halisidota caryae* Harris	Rare
	**Compsilura concinnata* Meig.	*Halisidota tessellaris* S. & A.	Rare
Porthetria dispar L., and *Euproctis chrysorrhoea* L.	**Compsilura concinnata* Meig.	*Melalopha inclusa* Hübn.	Rare

PREDATORS

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
Porthetria dispar L., and *Euproctis chrysorrhoea* L.	*Calosoma sycophanta* L.	*Porthetria dispar* L.	Abundant in sections of New England where oak trees abound that are heavily infested with this host. *Important enemy*
	Calosoma sycophanta L.	*Euproctis chrysorrhoea* L.	Common
	Calosoma sycophanta L.	*Stilpnotia salicis* L.	Apparently important
	Calosoma sycophanta L.	*Heterocampa guttivitta* Walk.	Rare
	Calosoma sycophanta L.	*Hemerocampa leucostigma* S. & A.	Rare
	Calosoma sycophanta L.	*Ennomos subsignarius* Hübn.	Common
	Calosoma sycophanta L.	Noctuid sp. (Tree climbing larvae.)	Common

PREDATORS (Cont'd)

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
	Calosoma sycophanta L.	*Malacosoma americana* Fab.	Rare
	Calosoma sycophanta L.	*Malacosoma disstria* Hübn.	Common
	Calosoma sycophanta L.	*Geometrid* sp.	Common

This predator does not refuse or hesitate to attack any species of Lepidopterous larvae ordinarily met on tree growth during the summer months at the time of its abundance; hence there are many other species that fall prey to it, the names of which are not yet recorded. Several species of Lepidopterous larvae have been offered as food to the beetles in jars and cages which were readily consumed by them but are not recorded above.

PREDATORS (Cont'd)

HOST FOR WHICH IMPORTED	NAME OF PARASITE	AMERICAN SPECIES ATTACKED	COMMENT ON ABUNDANCE
Porthetria dispar L., and *Euproctis chrysorrhoea* L.	*Carabus auratus* L.	Gray field slug (*Agriolimax agrestis* L.)	Apparently important
	Carabus auratus L.	Noctuid sp. (Ground cutworms)	Apparently important

This Carabid undoubtedly devours other species of soft-bodied insects ordinarily found in or near the ground.

Of these, it will be seen that the Tachinid fly *Compsilura concinnata* has readily accommodated itself to a number of native injurious species and that it is an introduction which is of great importance entirely aside from its efficiency as a parasite of the brown-tail moth.

It seems reasonably certain also that the little Braconid *Apanteles lacteicolor* is a species which will prove to have a very broad value. In addition to the records already given, I am informed by Mr. R. T. Webber of the Gipsy Moth Laboratory that the Tachinid mentioned is also probably a parasite on the larva of *Cimbex americana* (a sawfly) and on two other, unidentified sawflies; while Mr. C. W. Johnson, Curator of the Boston Society of Natural History, has noted an apparent parasitism of this species on the pine weevil (*Pissodes strobi*).

A most useful feature in the biology of the Tachinid fly is that with the larger caterpillar hosts the third stage of growth is the one in which they are nearly always attacked, which allows their Microgaster parasites to develop and emerge without having to struggle against a simultaneous parasitism by the Tachinid.

Moreover, this important imported Tachinid has not only spread all

over the whole gipsy moth territory in the United States but is found in some cases twenty miles beyond the line of gipsy moth spread.

I confess that I anticipate great benefits in this same broad way from *Apanteles lacteicolor* since it apparently attacks not only hairy caterpillars but naked caterpillars.

Of late the Bureau of Entomology has been introducing from south Europe the parasites of the imported European corn borer. Although it is too early to state positively that any of these imported parasites have been established, an important parasite known as *Habrobracon brevicornis*, found last summer in a region of south France near Hyères, has been brought over in very considerable numbers and has been reared in the laboratory with the utmost ease and some thousands have been liberated. It manages to lay its eggs in the larva of the borer while in the cornstalk, and, as it happens that we have a native cornstalk borer which is especially prevalent in the southeastern United States, we have experimented in the laboratory at Arlington, Massachusetts, with the latter species, and find that the European parasite takes to it readily and should become, if we can establish it, an important parasite of the southern corn pest. This work was done on the initiative of and under the direction of Mr. W. R. Walton.

Further than this, there exists a Lepidopterous borer in the sugar cane in Louisiana which is closely allied to the corn stalk-borer of the southern States and which will with little question be parasitized by this European parasite when they are once brought together. Inasmuch as the Habrobracon normally inhabits the Mediterranean littoral, it is altogether likely that it will accommodate itself to the climate of the Gulf of Mexico littoral more readily than it will the harsher climate of New England and New York. The experiment has actually been begun and it is found that the European parasite breeds freely in confinement at New Orleans on the sugar-cane borer. It is planned to liberate large numbers the present season on an isolated infested plantation in Louisana. We have strong hopes of its successful establishment.

The evolution of parasitism with insects is an interesting study. As in so many other directions in biological studies, we can trace the course of probable evolution by different stages which exist at the present time. The very catholic methods of the Tachinid fly listed above, which lays its eggs not only on all sorts of caterpillars, whether hairy or naked or large or small, but will even oviposit on the caterpillar-like sawfly larvae, and even upon a weevil grub in one instance (not, however, as yet fully verified), indicate rather surely that the parasitic habit has more recently become adopted with the Tachinids than with very many of the parasitic Hymenoptera.

But with the Hymenoptera we have all stages in the evolution, from

general parasitism to a highly specific parasitism, in the latter cases the parasite having become so perfectly modified to fit itself to the one species of host that it must mean an advanced stage of evolution.

In considering the importation of parasite species we must aim to have both kinds if they exist—the specifically constituted parasites and the general parasites. From the ranks of the latter we will have great help in the destruction of large numbers of the particular insect against which it was imported, but it will have the additional advantage of accommodating itself to other injurious species, and hence on the whole will probably be a more desirable addition to the American parasite fauna.

Although not parasites, mention should be made of two species of ground-beetles brought over from Europe in the course of the gipsy moth work. *Calosoma sycophanta* has been found to attack a number of injurious native caterpillars; and *Carabus auratus*, while probably primarily a feeder upon slugs, as it is in its native home, also feeds upon cutworms and probably other soft-bodied insects ordinarily found in or near the ground.

ON THE LOCATION OF THE ROOTS OF THE DERIVATIVE OF A POLYNOMIAL

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This note considers some geometric aspects of the problem of the approximate location of the roots of the derivative $f'(z)$ of a polynomial $f(z)$ when the roots of $f(z)$ are known. If $f(z)$ has m_1 roots in a circle C_1, m_2 roots in a circle C_2, and no other roots, the roots of $f'(z)$ are known to lie in C_1, C_2, and a certain third circle C which is readily determined.[1] Moreover, if $f(z)$ has m_1 roots in C_1, m_2 roots in C_2, m_3 roots in C_3, and no other roots, and if the circles C_1, C_2, C_3, are equal and have collinear centers, the roots of $f'(z)$ are known to lie in those circles and in two circles C', C'' equal to the original circles and whose centers are collinear with their centers.[2] In each of these cases, the actual *locus* of the roots of $f'(z)$ consists of the circles stated, when the given circles are the loci of those roots of $f(z)$ (supposed to vary independently) which they contain.

The second of these results suggests the problem of the determination of the locus of the roots of $f'(z)$ when the locus of the roots of $f(z)$ consists of three circles C_1, C_2, C_3 which are not supposed equal and with collinear centers.[3] The solution of that problem is indicated in the present note; the answer is stated in the

THEOREM. *Let circles C_1, C_2, C_3 be the respective loci of m_1, m_2, m_3 roots of a polynomial $f(z)$. Then the locus of the roots of $f'(z)$ consists of C_1, C_2,*

C_3 (except that C_i is to be omitted if $m_i = 1$) and of a region R bounded by the bicircular quartic

$$
\begin{aligned}
&m_1^2[(x-b_1)^2 + (y-b_2)^2 - r_2^2]\,[(x-c_1)^2 + (y-c_2)^2 - r_3^2] \\
&+m_2^2[(x-a_1)^2 + (y-a_2)^2 - r_1^2]\,[(x-c_1)^2 + (y-c_2)^2 - r_3^2] \quad \times \\
&+m_3^2[(x-a_1)^2 + (y-a_2)^2 - r_1^2]\,[(x-b_1)^2 + (y-b_2)^2 - r_2^2] \\
(1)\quad &+2m_2m_3[(x-a_1)^2 + (y-a_2)^2 - r_1^2]\,[(x-b_1)(x-c_1) + (y-b_2)(y-c_2) - r_2r_3] \\
&+2m_1m_3[(x-b_1)^2 + (y-b_2)^2 - r_2^2]\,[(x-a_1)(x-c_1) + (y-a_2)(y-c_2) - r_1r_3] \\
&2m_1m_2[(x-c_1)^2 + (y-c_2)^2 - r_3^2]\,[(x-a_1)(x-b_1) + (y-a_2)(y-b_2) - r_1r_2] = 0;
\end{aligned}
$$

here the circles C_1, C_2, C_3 have the respective centers and radii: (a_1, a_2), r^1; (b_1, b_2), r_2; (c_1, c_2), r_3. If the curve (1) consists of two nested ovals, R is the region bounded by the outer oval; if (1) does not consist of two nested ovals, R is the interior of the curve.

If no two of the regions C_1, C_2, C_3, R have any point in common, they contain respectively $m_1 - 1$, $m_2 - 1$, $m_3 - 1$, 2 roots of $f'(z)$. If R is in two parts, which have no point in common with each other or any of the regions C_i which is a part of the locus of the roots of the derivative, each part of R contains one root of $f'(z)$.

This theorem replaces the problem of the exact location of the roots of $f'(z)$, which depends on the solution of an equation of degree $m_1 + m_2 + m_3 - 1$, by the problem of their approximation which is solved by plotting a quartic curve, which depends on the solution of equations of degree 4. In particular cases—such as that mentioned in the first paragraph of this note—the quartic (1) degenerates, in which case the new problem is still simpler, for it depends merely on the solution of quadratics.

The theorem can be proved by a straightforward method which we now indicate. The locus of the roots of $f'(z)$ is readily found when C_1 is not a null-circle but C_2 and C_3 are both null-circles; every point of the boundary is a point of the curve (1) where $r_2 = r_3 = 0$. The locus of the roots of $f'(z)$ can then be found when neither C_1 nor C_2 is a null-circle by determining the envelope of the former boundary; every point of the new boundary is a point of (1) where $r_3 = 0$. Finally the locus is found in the general case by determining the envelope of this last determined boundary; every point of the boundary is a point of the curve (1).

A particularly interesting case of the theorem occurs if $m_1 = m_2 = m_3$, $r_1 = r_2 = r_3$, and the centers of C_1, C_2, C_3 are the vertices of an equilateral triangle. The locus of the roots of $f'(z)$ is (in addition to the regions C_i) the interior and boundary of a circle whose center is the center of this triangle and whose radius is $(r_1^2 + r_1h)^{1/2}$, h being the distance from the center of the triangle to the centers of the circles C_i.

The geometry of this entire situation deserves further inquiry, with especial reference to the cases where (1) degenerates. Our theorem generalizes presumably to the case in which any number of circles C_i form the

loci of the roots of $f(z)$, and also where we consider not merely the first but the k^{th} derivative of $f(z)$. The writer hopes to continue the study of these problems.

[1] See paper by the writer, *Trans. Amer. Math. Soc.*, **22**, 1921 (101–116).

[2] See a forthcoming paper by the writer in the *Trans. Amer. Math. Soc.*; a similar result holds for any number of equal circles C_i which have collinear centers.

[3] This problem is set forth explicitly in the paper referred to in the preceding footnote.

ON THE DISTRIBUTION OF THE VELOCITIES OF STARS OF LATE TYPES OF SPECTRUM

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The recent determination of spectroscopic paralaxes or stars of the later types of spectrum made by Adams and his collaborators[1] at the Mt. Wilson Observatory has made it possible to compute the absolute motions in space of about 1300 stars of spectral types A_7 to M for which the proper motions and radial velocities are known. The spectroscopic parallaxes for which the errors are proportional to the parallaxes themselves are in general better adapted for this purpose than the trigonometric parallaxes since the accuracy of the smaller parallaxes is considerably greater.

The three velocity-components of these stars have been computed with reference to the galactic system of coördinates, the pole of galaxy being assumed to have the position A = 190.6°; D = + 27.2°. This system of coördinates is very convenient in the study of the systematic motions of the stars since these motions are mainly in the galactic plane.

Solar Motion.—The solar motion can be obtained directly by forming the algebraic mean of the velocity-components which are measured with reference to the sun. By taking the opposite vector we find the sun's velocity relative to the group of stars employed. The following values for the sun's velocity have been derived in this way.

SPECIAL TYPE	M	NO.	$A_\odot$	$D_\odot$	V KM.
A6 to F9	<3	153	269°	+27°	21.2
	>3	133	274	35	23.4
G0 to G9	<3	224	266	32	19.9
	>3	154	279	23	40.2
K0 to K9	<4	325	282	43	18.3
	>4	112	286	31	33.1
M	<4	98	271	44	16.2
A6 to M	Giants	800	273	37	18.8
A6 to M	Dwarfs	415	281	29	31.7

The stars of each spectral type have been divided into two groups according to their absolute magnitude M, and these groups, in the case of the later types at least, can be identified with the giant and dwarf classes. We see that for the giant stars the declination D_0 of the sun's apex shows a steady increase from 27° to 44° with advancing spectral type. The same result has been found by several investigators who have studied the distribution of proper motions, and the effect seems to be very marked among the apparently faint stars studied by Dyson and Thackeray.[2] This indicates a larger proportion of stars belonging to the second stream among the K and M stars than among those of early types. The dwarf stars show a much larger value of the velocity of the sun than that deduced from the stars in general.

A very remarkable feature is the behavior of the stars of high velocity. If we group the stars according to their velocities with reference to an origin corrected for the standard solar velocity of 20.0 km. directed toward the point A = 270°; D = + 30°, we find that the velocity of the sun increases remarkably with increasing speed of the stars relative to which its motion is referred.

The following table gives the result of such a computation, the stars being grouped according to velocity.

V KM./SEC.	A_0	D_0	V KM.
0 to 60.....	272°	+30°	20.6
60 to 100.....	295	+43	36.3
100 to 150.....	289	+39	76
>150.....	313	+54	209

We see from this comparison that the stars of high speed have a systematic motion which differs from that of stars of moderate speed, and that they have a tendency to move in a direction opposite to the sun's motion. The same result was indicated by Adams and Joy[3] who found from a study of the space-motions of 37 stars of high radial velocity that nearly all the apices were confined to one hemisphere.

Distribution of Velocities.—The peculiarities in the motions of the stars can best be studied by determining the frequency-function according to which the velocity-vectors are distributed. Two different types of frequency-functions have been used hitherto, namely, those based upon the two-drift theory of Kapteyn[4] and the ellipsoidal theory of Schwarzschild.[5] If use is made of three-dimensional velocity-vectors it is easy to decide between these two theories, but in order to represent the actual distribution of the velocity-vectors in all its aspects a more general type of frequency-function is necessary. In the first place the velocity-vectors were all corrected for a standard solar velocity of 20.0 km. in the direction $A_0 = 270°$ and $D_0 = + 30°$. About 100 stars which had been selected for observa-

tion on account of their small proper motions were omitted since they would tend to give a "star stream nearly parallel to the direction of the sun's motion. Stars recognized as members of the Taurus Group were also omitted, as well as fainter components of double stars.

The frequency-function used is a three dimensional cosine-series with the origin in the center of the limiting planes. Such a function can be written

$$F(xyz) = a_0 + a_{100}\sin\alpha + a_{010}\sin\beta + a_{001}\sin\gamma + a_{200}\cos 2\alpha + a_{110}\sin\alpha\sin\beta + a_{101}\sin\alpha\sin\gamma + a_{020}\cos 2\beta + a_{011}\sin\beta\sin\gamma + a_{002}\cos 2\gamma + a_{300}\sin 3\alpha + a_{210}\cos 2\alpha\sin\beta + a_{201}\cos 2\alpha\sin\gamma + \ldots.$$

$$\alpha = \frac{\pi x}{2c_1} \qquad \beta = \frac{\pi y}{2c_2} \qquad \gamma = \frac{\pi z}{2c_3}$$

In these expressions $\pm c_1$, $\pm c_2$ and $\pm c_3$, are the six limiting planes within which the function can be regarded as an interpolation formula in three dimensions. For different values of the density F we obtain a sequence of equifrequential surfaces. We see then that α, β, and γ vary between the limits $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$. The integration between certain limits as well as the determination of the coefficients in the series can be performed easily. All terms up to the sixth order have been computed, while in the case of the dwarf stars, where the limits are very large, terms up to the 10th order have been included. The direction of the x-axis is towards the intersection of the galactic plane with the equator, (in Aquila), the y axis is in the direction of galactic longitude 90°, and the z axis towards the galactic north pole.

After the coefficients in the trigonometric series had been computed, a synthesis was made for $z = 0$ and $y = 0$. Curves of equal frequency were then drawn in the xy plane and the xz plane. These curves are the intersections of equifrequential surfaces with these two planes. The total number of stars including those outside the limits have in all cases been reduced to 1000.

The general appearance of the frequency surfaces can be easily seen from the diagrams in figure 1, which give the intersections of the former with the xy and xz planes for the brighter stars of spectral types F to G, and for the dwarf stars of types G0 to M. The curves for the brighter K and M stars are similar to those for the types G_8 to K_1, although the curves are more nearly circular. The crosses (+) represent the velocities of stars belonging to the Taurus Group which were omitted in the analysis. The symbol ⊙ represents the velocity of the sun, whose velocity components in our system of coördinates are $x = +17.0$; $y = +7.4$; $z = +7.4$ km./sec.

The general conclusions to be drawn from a study of these diagrams and the series on which they are based are the following:

1. The giant stars of all types form one single "stream" with an ellip-

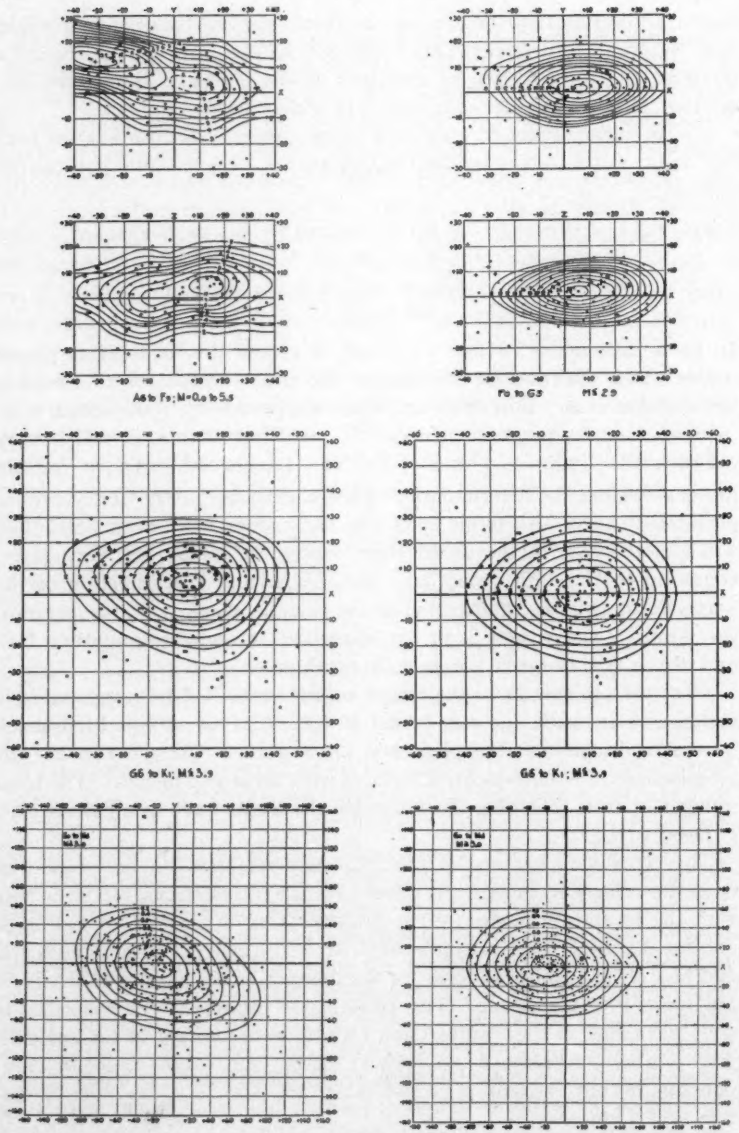

FIG. 1

Distribution of velocities of stars showing the intersections of equi-frequential surfaces of the terminal points of the velocity-vectors with the xy and xz planes.

soidal distribution; the longest and the intermediate axes in all cases lying very nearly in the galactic plane and coinciding approximately with the x and y axes. The shortest axis is perpendicular to the galactic plane. The elongation of the ellipsoids is largest for the bright F stars and decreases for later types, the distribution being nearly spherical for the M stars.

2. The most frequent velocity does not coincide with the origin but lies in all cases in the first quadrant of the xy plane, and has the approximate values $x = +5.6$; $y = +2.3$; $z = +1.6$ km./sec. Referring the sun's motion to this origin we find its velocity to be 13.8 km./sec. towards the apex $A_0 = 267°$; $D_0 = +32°$.

3. If we exclude the very brightest F stars, many of which are Cepheid variables and stars of similar spectra, and take together stars of types A_7 to F_9 of absolute magnitude 0 to 6, we find that these stars divide themselves into two streams. One of these coincides with the general group of giant stars, as can be seen from the agreement of its condensation-points with that of the giant stars of other spectral types while the other stream moves nearly parallel to the Taurus Group. This group[6] is moving towards the point $\alpha = 92°$; $\delta = +7°$ with a velocity of 44 km./sec. or in galactic coördinates with our adopted origin $x = -26$, $y = +15$, $z = +4$ km./sec. This stream comprises the majority of the stars of F type and its existence seems to indicate that a large number of these stars in all regions of the sky are traveling with a velocity nearly identical with that of the Taurus Group. Even among the brightest F stars as well as the G-type giants the presence of members of this group is indicated by the protruding form of the equifrequential surfaces in this region. This stream might be identified with Kapteyn's First Drift although the galactic latitude is not zero.

4. The dwarf stars among which have been included those with absolute magnitudes fainter than 3.0 seem to form a group of their own as regards their motions. The distribution of the velocities is ellipsoidal, the major, intermediate and shortest axes having the galactic longitudes 162°, 70° and 291° and latitudes +8°, +9° and +77°, respectively. The most frequent velocity is $x = -19$; $y = -2$; $z = +1$ km./sec. which nearly coincides with Kapteyn's First Drift, while his Second Drift has been swallowed up by the ellipsoidal stream of giant stars.

There is a very marked asymmetry in the distribution of the velocities around the most frequent velocity-vector. The stars of high velocity show a tendency to avoid the first quadrant of the xy plane. This asymmetry produces a systematic difference between the most frequent velocity and the mean of the velocity-components (the centroid). If we omit the stars of most rapid motion the centroid falls very near the origin and gives us nearly the standard solar velocity. That this asymmetry is not due to the selection of stars of large proper motion moving opposite to the sun is

shown by a study of the velocity-distribution of stars having very high speeds. In the diagram in figure 2 are plotted the projections in the xy plane of the velocities of stars having space motions larger than 100 km./sec. The effect of the selection of proper motions relative to the sun instead of to our adopted origin is then insignificant, and we find that whatever the spectral types or the absolute magnitude, if the velocity of the star

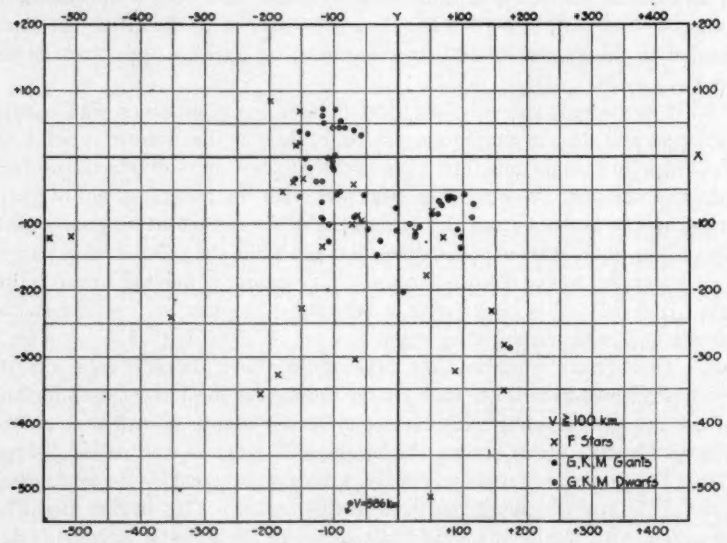

FIG. 2

Projections in the galactic plane of the velocities of stars having a speed greater than 100 km. per second.

is great, there is an avoidance of the first quadrant in the xy plane. No star of speed higher than 100 km./sec. is moving in this direction, but a large proportion of them are moving toward the third quadrant, a result in agreement with that found by Adams and Joy[3] for stars of high radial velocity.

[1] *Mt. Wilson Contr.* No. 199, *Astroph. J., Chicago,* **53,** 1921 (13).

[2] *London, Mon. Not. R. Astron. Soc.,* **79,** 1919 (201).

[3] *Mt. Wilson Contr.* No. 163, *Astroph. J.,* **49,** 1919 (179).

[4] Kapteyn, Address before St. Louis Exposition Congress, 1904.

[5] Schwarzschild, *Göttingen Nachrichten,* 1907, p. 614. A summary of the Theories is given in Eddington's, *Stellar Movements and the Structure of the Universe,* London, 1914.

[6] Hertzsprung, *Astron. Nachrichten.* **209,** 1918.

CONTRIBUTION TO THE ENERGETICS OF EVOLUTION*

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It has been pointed out by Boltzmann[1] that the fundamental object of contention in the life-struggle, in the evolution of the organic world, is available energy.[2] In accord with this observation is the principle[3] that, in the struggle for existence, the advantage must go to those organisms whose energy-capturing devices are most efficient[4] in directing available energy into channels favorable to the preservation of the species.

The first effect of natural selection thus operating upon competing species will be to give relative preponderance (in number or mass) to those most efficient in guiding available energy in the manner indicated. Primarily the *path* of the energy flux through the system will be affected.

But the species possessing superior energy-capturing and directing devices may accomplish something more than merely to divert to its own advantage energy for which others are competing with it. If sources are presented, capable of supplying available energy in excess of that actually being tapped by the entire system of living organisms, then an opportunity is furnished for suitably constituted organisms to enlarge the total energy flux[5] through the system. Whenever such organisms arise, natural selection will operate to preserve and increase them. The result, in this case, is not a mere diversion of the energy flux through the system of organic nature along a new path, but an increase of the total flux through that system.

Again, so long as sources exist, capable of supplying matter, of a character suitable for the compositon of living organisms, in excess of that actually embodied in the system of organic nature, so long is opportunity furnished for suitably constituted organisms to enlarge the total mass of the system of organic nature. Whenever such organisms arise, natural selection will operate to preserve and increase them, provided always that there is presented a residue of untapped available energy. The result will be to increase the total mass of the system, and, with this total mass, also the total energy flux through the system, since, other things equal, this energy flux is proportional to the mass of the system.

Where a limit, either constant or slowly changing,[6] is imposed upon the total mass available for the operation of life processes, the available energy per unit of time (available power) placed at the disposal of the organisms, for application to their life tasks and contests, may be capable of increase by increasing the rate of turnover of the organic matter through the life cycle. So, for example, under present conditions,[7] the United States produce annually a crop of primary and secondary food amounting to

about 1.37×10^{14} kilogramcalories per annum, enough to support a population of about 105 million persons (equivalent to about 88 million adults) at the present rate of food consumption (4,270 kilogram-calories per adult per day). Suppose, as a simple, though rather extreme illustration, that man found means of doubling the rate of growth of crops, and of growing two crops a year instead of one. Then, without changing the average crop actually standing on the fields, the land would be capable of supporting double the present population. If this population were attained, the energy flux through the system composed of the human population and the organisms upon which it is dependent for food, would also be doubled. This result would be attained, not by doubling the mass of the system (for the matter locked up in crops, etc., at a given moment would be, on an average, unchanged) but by increasing the velocity of circulation of mass through the life cycle in the system. Once more it is evident that, whenever a group of[8] organisms arises which is so constituted as to increase the rate of circulation of matter through the system in the manner exemplified, natural selection will operate to preserve and increase such a group, provided always that there is presented a residue of untapped available energy, and, where circumstances require it, also a residue of mass suitable for the composition of living matter.

To recapitulate: In every instance considered, natural selection will so operate as to increase the total mass of the organic system, to increase the rate of circulation of matter through the system, and to increase the total energy flux through the system, so long as there is presented an unutilized residue of matter and available energy.

This may be expressed by saying that *natural selection* tends to make the energy flux through the system a maximum, so far as compatible with the constraints to which the system is subject.

It is not lawful to infer immediately that *evolution* tends thus to make this energy flux a maximum. For in evolution two kinds of influences are at work: selecting influences, and generating influences. The former select, the latter furnish the material for selection.

If the material furnished for selection is strictly limited, as in the case of a simple chemical reaction,[9] which gives rise to a finite number of products, the range of operation of the selective influences is equally limited.

In the case of organic evolution the situation is very different. We have no reason to suppose that there is any finite limit to the number of possible types of organisms. In the present state of our knowledge, or rather our ignorance, regarding the generating influences that furnish material for natural selection, for organic evolution, an element of uncertainty enters here. It appears, however, at least *a priori* probable that, among the certainly very large (if not infinite) variety of types presented for selection, sooner or later those will occur which give the opportunity for

selection to operate in the direction indicated, namely so as to increase the total mass of the system, the rate of circulation of mass through the system, and the total energy flux through the system. If this condition is satisfied, the law of selection becomes also the law of evolution:

Evolution, in these circumstances, proceeds in such direction as to make the total energy flux through the system a maximum compatible with the constraints.

We have thus derived, upon a deductive basis, at least a preliminary answer to a question proposed by the writer in a previous publication.[10] It was there pointed out that the influence of man, as the most successful species in the competitive struggle, seems to have been to accelerate the circulation of matter through the life cycle, both by "enlarging the wheel," and by causing it to "spin faster." The question was raised whether, in this, man has been unconsciously fulfilling a law of nature, according to which some physical quantity in the system tends toward a maximum. This is now made to appear probable; and it is found that the physical quantity in question is of the dimensions of power, or energy per unit time, as was hinted by the writer on an earlier occasion.[11]

It may be remarked that the principle of maximum energy flux here set forth bears a certain outward resemblance to a principle enunciated by Ostwald:[12] "Of all possible energy transformations, that one takes place, which brings about the maximum transformation in a given time." This principle of Ostwald's, however, is based on entirely different grounds from those here brought forward. It is not of general applicability, and in particular, its application to systems of the kind here considered does not appear warranted.

Addendum. Since the paragraphs above were penned, the writer has received from the booksellers a copy of Professor J. Johnstone's book, "The Mechanism of Life" (1921), in which (pp. 217–221) that author touches on matters closely related to those here discussed. Professor Johnstone draws, however, a somewhat different conclusion, namely that "In living processes the increase of entropy is retarded."[13] He points out that this is true, primarily, of plants; but that among animals[14] also natural selection must work toward the weeding out of unnecessary and wasteful activities, and thus toward the conserving of free energy, or, what amounts to the same thing, toward retarding energy dissipation.

This is perhaps not wholly convincing, for the first effect of the advent of animal organisms in a world peopled with a purely vegetable population, would certainly seem to be an acceleration of the process of dissipation. It appears, therefore, that at certain stages in the evolution of the system, at the least, life must have tended to increase rather than decrease dissipation. And even if animals ultimately evolve in the direction of decreased dissipative effect, they still remain essentially a dissipative type,

as compared with plants, and, to make Professor Johnstone's argument conclusive, it would seem necessary to show, not merely that the animal organism evolves in that direction, but that the system of coupled transformers, plant and animal, as a whole has so evolved.

Professor Johnstone's conclusion is, however, not essentially incompatible with the one developed in this paper. Where the supply of available energy is limited, the advantage will go to that organism which is most efficient, most economical, in applying to preservative uses such energy as it captures. Where the energy supply is capable of expansion, efficiency or economy, though still an advantage, is only one way of meeting the situation, and, so long as there remains an unutilized margin of available energy, sooner or later the battle, presumably, will be between two groups or species equally efficient, equally economical, but the one more apt than the other in tapping previously unutilized sources of available energy.

There is here a problem that will call for further investigation. In particular, it remains to be established just what is the significance of the phrase "compatible with the constraints" which, in the presentation here given, modifies the maximum principle enunciated. The present communication is intended rather a preliminary than as an attempt to say the last word on the subject. More detailed discussion is planned for another occasion.

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[1] *Der zweite Hauptsatz der mechanischen Wärmetheorie*, 1886 (Gerold, Vienna), p. 21; *Populäre Schriften*, No. 3, Leipsic, 1905; Nernst, *Theoretische Chemie*, 1913, p. 819; Burns and Paton, *Biophysics*, 1921, p. 8, H. F. Osborn, The Origin and Evolution of Life, 1918, p. XV.

[2] Compare also Sir Oliver Lodge, *Life and Matter*, 1906, pp. 139, 140.

[3] Lotka, A. J., *Ann. Naturphil*, 1910, pp. 67, 68; *Proc. Nat. Acad., Sci.*, 1921, pp. 194, 195.

[4] Lotka, A. J., *Proc. Washington Acad.*, **5**, 1915, pp. 360, 397.

[5] The term *energy flux* is here used to denote the available energy absorbed by and dissipated within the system per unit of time.

[6] As, for example, if the total mass of the system is capable of accretion, but only at a limited velocity, in which case the phenomenon of a moving equilibrium may present itself. Compare Lotka, A. J., *Proc. Nat. Acad. Sci.*, **7**, 1921, p. 168.

[7] Pearl, R., *The Nation's Food*, 1920, pp. 218, 203, 258, 80, 245; *Amer. J. Hygiene*, **1**, 1921, p. 598.

[8] Owing to the fact that in existing organisms the anabiotic and catabiotic functions are very largely segregated in different types (plants and animals), evolution will here operate upon systems or groups of at least two species, one species of autotrophic ana bions, and one of heterotrophic catabions.

[9] Compare Lotka, A. J., *Amer. J. Sci.*, **24**, 1907, pp. 204, 216.

[10] Lotka, A. J., *Proc. Nat. Acad. Sci.*, **7**, 1921, p. 172.

[11] *Idem., Ann. Naturphil.*, 1910, p. 70. It is there suggested that the continuous energy transformations associated with the maintenance of a steady state would probably be found to play the dominant rôle, while any "latent heat" effect associated

with a change in the distribution of matter among the several species composing the system, would probably play a subordinate rôle; in contrast with the condition of affairs familiar in ordinary physico-chemical systems. This is an obvious inference from the observation that the several species of organisms are distinguished much more by structural differences than by differences in chemical composition.

[12] Ostwald, W., *Lehrbuch der allgemeinen Chemie*, 1892, vol. 2, p. 37; Siebel, J. E *Compend of Mechanical Refrigeration*, 1915, p. 88. For a discussion of the validity and limitations of Ostwald's principle see Helm G., *Die Energetik*, 1898, pp. 248; Neumann, C., *Leipziger Berichte*, 1892, p. 184.

[13] That living organisms may be capable of retarding the energy flux through the system of nature was suggested by the present writer in *Ann. Naturphil.*, 1910, p. 60.

[14] Johnstone, J., *The Mechanism of Life*, 1921, p. 220.

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*NATURAL SELECTION AS A PHYSICAL PRINCIPLE**

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In a paper presented concurrently with this, the principle of natural selection, or of the survival of the fittest (persistence of stable forms), is employed as an instrument for drawing certain conclusions regarding the energetics of a system in evolution.

Aside from such interest as attaches to the conclusions reached, the method itself of the argument presents a feature that deserves special note. The principle of natural selection reveals itself as capable of yielding information which the first and second laws of thermodynamics are not competent to furnish.

The two fundamental laws of thermodynamics are, of course, insufficient to determine the course of events in a physical system. They tell us that certain things cannot happen, but they do not tell us what does happen.

In the freedom which is thus left, certain writers have seen the opportunity for the interference of life and conciousness in the history of a physical system. So W. Ostwald[2] observes that "the organism utilizes, in manyfold ways, the freedom of choice among reaction velocities, through the influence of catalytic substances, to satisfy advantageously its energy requirements." Sir Oliver Lodge also, has drawn attention to the guidance[3] exercised by life and mind upon physical events, within the limits imposed by the requirements of available[4] energy. H. Guilleminot[5] sees the influence of life upon physical systems in the substitution of guidance by choice in place of fortuitous happenings, where Carnot's principle leaves the course of events indeterminate. As to this, it may be objected that the attribute of fortuitousness is not an objective quality of a given event. It is the expression of our subjective ignorance, our lack of com-

plete information, or else our deliberate ignoring of some of the factors that actually do determine the course of events. Admitting, however, broadly, the directing influence of life upon the world's events, within the limits imposed by the Mayer-Joule and the Carnot-Clausius principles, it would be an error to suppose that the faculty of guidance which the established laws of thermodynamics thus leave open, is a peculiar prerogative of living organisms. If these laws do not fully define the course of events, this does not necessarily mean that this course, in nature, is actually indeterminate, and requires, or even allows, some extra-physical influence to decide happenings. It merely means that the laws, as formulated, take account of certain factors only, leaving others out of consideration; and that the data thus furnished are insufficient to yield an unambiguous answer to our enquiry regarding the course of events in a physical system. Whether life is present or not, something more than the first and second laws of thermodynamics is required to predict the course of events. And, whether life is present or not, *something* definite *does happen*, the course of events is determinate, though not in terms of the first and second laws alone. The "freedom" of which living organisms avail themselves *under the laws of thermodynamics* is not a freedom in fact, but a spurious freedom[6] arising out of an incomplete statement of the physical laws applicable to the case. The strength of Carnot's principle is also its weakness: it holds true independently of the particular mechanism or configuration of the energy transformer (engine) to which it is applied; but, for that very reason it is also incompetent to yield essential information regarding the influence of mechanism upon the course of events. In the *ideal* case of a reversible heat engine the efficiency is independent of the mechanism. *Real* phenomena are irreversible; and, in particular, trigger action,[7] which plays so important a rôle in life processes, is a typically irreversible process, the release of available energy from a "false" equilibrium. Here mechanism is all-important. To deal with problems presented in these cases requires new methods,[8] requires the introduction, into the argument, of new principles. And a principle competent to extend our systematic knowledge in this field seems to be found in the principle of natural selection, the principle of the survival of the fittest, or, to speak in terms freed from biological implications, the principle of the persistence of stable forms.

For the battle array of organic evolution is presented to our view as an assembly of armies of energy transformers—accumulators (plants), and engines (animals); armies composed of multitudes of similar units, the individual organisms. The similarity of the units invites statistical treatment, the development of a statistical mechanics of which the units shall be, not simple *material particles* in ordinary reversible collision of the type familiar in the kinetic theory, collisions in which action and reac-

tion were equal; the units in the new statistical mechanics will be *energy transformers* subject to irreversible collisions of peculiar type—collisions in which trigger action is a dominant feature:

When the beast of prey A sights its quarry B, the latter may be said to enter the field of influence of A, and, in that sense, to collide with A. The energy that enters the eye of A in these circumstances may be insignificant, but it is enough to work the relay, to release the energy for the fatal encounter. And because evolution works with armies built up of similar units, the seemingly erratic workings of the relay mechanism (in which action and reaction are not equal, and seem subject to no simple general law) are not, in effect, erratic, but range themselves according to law and order, for those species of units, those types of transformers, are picked out for survival, whose mechanism possesses certain definite properties. Thus the principle of natural selection makes its entry into dynamics.

Further elaboration of these concepts must be reserved for a future occasion.

In systems evolving toward a true equilibrium (such as thermally and mechanically isolated systems, or the isothermal systems of physical chemistry), the first and second laws of thermodynamics suffice to determinate at any rate the end state; this is, for example, independent of the amount of any purely catalytic substance that may be present. The first and the second law here themselves function as the laws of selection and evolution, as has been recognized by Perrin[9] and others, and exemplified in some detail by the writer, for the case of a monomolecular reversible reaction.[10]

But systems receiving a steady supply of available energy (such as the earth illuminated by the sun), and evolving, not toward a true equilibrium, but (probably) toward a stationary state, the laws of thermodynamics are no longer sufficient to determine the end state; a catalyst, in general, does affect the final steady state. Here selection may operate not only among components taking part in transformations, but also upon catalysts, in particular upon auto-catalytic or auto-catakinetic constituents of the system. Such auto-catakinetic constituents are the living organisms,[11] and to them, therefore the principles here discussed, apply.

That the principle of selection is competent to yield information beyond the scope of the laws of thermodynamics has been very clearly set forth, independently, by H. Guilleminot.[12] The present writer has long realized that the principle is capable of such application; that it functions, as it were, as a third law of thermodynamics (or a fourth, if the third place be given to the Nernst principle). If he has not, before this date, explicitly stated the case, this is mainly because his writings have followed a definite, systematic plan, announced in his early publications.[13] Viewing evolution as a change in the distribution of matter among the components of a physical system, the study of evolution naturally di-

vides itself into two fields. The one, which might be termed the *stoichiometry* of evolution, deals with mass relations: the relative amounts of the different species of matter present, and the changes in these amounts; the *kinetics* of evolution. The second field of study is the *dynamics* or *energetics* of evolution, the scope of which is sufficiently indicated by these terms. It appeared desirable to lay the foundation of the first, as the more elementary, of these fields, before proceeding to a systematic exposition of the second. This is the plan which has been closely followed by the writer in the past, and which it is hoped to develop in greater completeness in the future. Material held in reserve, and relating to the dynamics of evolution, will then be brought forward in its proper place. The present issue of this advance sheet is prompted by a recent reading of Guilleminot's book, which, through a series of mishaps, has only recently come to the writer's hand.

*Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 60.

[1] These PROCEEDINGS, **8** p. 147.

[2] Ostwald, W., *Vorlesungen über Naturphilosophie*, 1902, p. 328.

[3] Sir Oliver Lodge, *Life and Matter*, 1906, p. 144.

[4] *Ibid.*, pp. 148, 149; *Nature*, **67**, p. 595; **68**, p. 31.

[5] Guilleminot, H., *La Matière et la Vie*, 1919, p. 121, et passim.

[6] This remark must be understood to apply only to that freedom which arises out of the incompleteness of the first and second law as determinants of the course of events. The writer does not here take sides, one way or another, in the controversies regarding free will, determinism, vitalism as distinguished from mechanistic conceptions, etc.

[7] Compare Lotka, A. J., *J. Washington Acad. Sci.*, **2**, 1912, p.7 1; Guilleminot, H., *loc. cit.*, p. 115; Johnstone, J., *The Mechanism of Life*, 1921, p. 49.

[8] Lotka, A. J., *Proc. Nat. Acad. Sci.*, **7**, 1921, pp. 194, 196.

[9] Perrin, J., *Traité de Chimie*, 1903, vol. 1, pp. 142–143; Chowlson, O. D., *Lehrbuch der Physik.*, 1905, vol. 3, p. 499; *Scientia*, **3**, 1910, p. 51; Lotka, A. J., *Science Progress*, **14**, 1920, p. 406.

[10] For recent substantiation of some of the details of the presentation there given see Baly, E. C. C., *Nature*, vol. 109, 1922, p. 344.

[11] D'Arcy Thompson (*Growth and Form*, 1917, p. 132) attributes the origination of this concept to Chodat, quoted by Monnier, A. (*Publ. Inst. Bot. Univ. Genève*, (7) **III**, 1905. There seem to be, however, some earlier indications of the same thought. The following bibliography, which makes no pretense of completeness, is culled from works ready at hand: Errera, L., *Revue de l' Université de Bruxelles*, **5**, 1899–1900. May issue; Ostwald, W., *Vorlesungen über Naturphilosophie*, 1902, pp. 342, et seq.; Bastian, H. C., *The Nature and Origin of Living Matter*, 1905, p. 46, et seq.; Robertson, T. B., *Arch. Entwickelungsmechanik Org.*, **25**, 1908, p. 581, **25**, p. 108; Ostwald, Wo., *Die zeitlichen Eigenschaften der Entwickelungsvorgänge*, 1908; Hatai, S., *Anat. Rec.*, **5**, 1911, p. 373; Enriques, *Biol. Centralbl.*, 1909, p. 337; Lotka, A. J., *Z. physik. Chem.*, **72**, 1910, p. 511; **80**, 1912, p. 159; *J. Phys. Chem.*, **14**, 1910, p. 274; *Proc. Nat. Acad. Sci.*, **6**, 1920, p. 275; Pearl, R., *Amer. J. Hygiene*, **1**, 1921, p. 592.

[12] Guilleminot, H., *loc. cit.*, p. 118, 154, et passim.

[13] Lotka, A. J., *Am. J. Sci.*, **24**, 1907, p. 216; *Ann. Naturphil.*, 1910, p. 74.